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Let's Regress...

by James Walker

Regression analysis is a useful tool to quantify the common behavior of marginal productivity or decreasing returns, which is demonstrated everywhere in real estate. The computers and software used today make regression analysis easy and have removed the complexity normally associated with this technique. This article presents a procedure to easily apply regression analysis to produce more accurate appraisals and advocates the general use of regression in all appraisals.

We've all struggled with size adjustments for a 4,400 square foot subject when we only have 2,000 SF and 7,500 SF comps – how much do we adjust for square footage? We know the more area we have, the less each unit is worth, but how should these adjustments be made? How can we improve the accuracy of the appraisal? Regression analysis is the key to using dissimilar comparables with accuracy.

Decreasing contribution, or marginal productivity, is a typical behavior for most data encountered in real estate. It's relatively easy to tame this behavior and increase the accuracy of our appraisals by applying regression analysis with existing computers and software.

The usefulness and appropriateness of regression in appraisals is well documented in recent appraisal literature.[1,2,3] Regression techniques have been refined[4,5] and are less complicated[6,7] to use. This article, which demonstrates how easy and accurate regression can be, extends these finding to advocate the universal use of regression analysis in all appraisal work.

Market Behavior

It is common sense to appraisers that the more of a given quantity we have, the less each unit is worth. The value per acre on a 2 acre parcel is much higher than the value per acre on a 10 acre parcel, everything else being generally equal. Similarly, the value per square foot for a 10,000 SF office is much higher than for a 100,000 SF office.
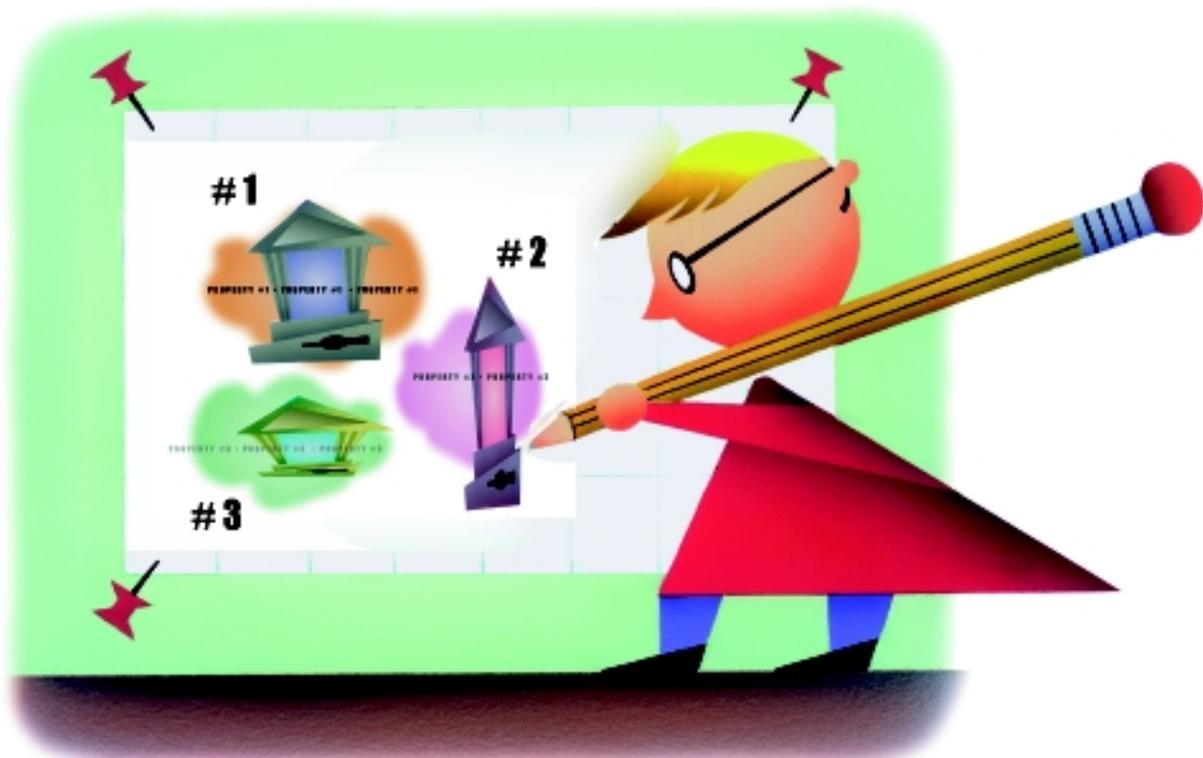
As an example, Grid 1 (right) presents land sales from the same town in a one-year period.

Sales are arranged in increasing order of parcel size. Note that the value per acre in general decreases as the parcel size increases.

Constructing a graph (Graph 1, right) of this behavior, the value per acre, is plotted against the parcel size. We can even let the computer calculate and overlay a curve of this relationship.[8]

The graph clearly shows the decreasing relationship of value per acre to parcel size and shows what we intuitively know. There is an easy procedure to determine the formula of the curve and apply it in our appraisals. But first, what is the basis in theory for this behavior?

Principles that Apply

This behavior is called the "law of decreasing returns" or the "principle of diminishing marginal productivity" [9] and has its theoretical basis in the principle of contribution. [10]
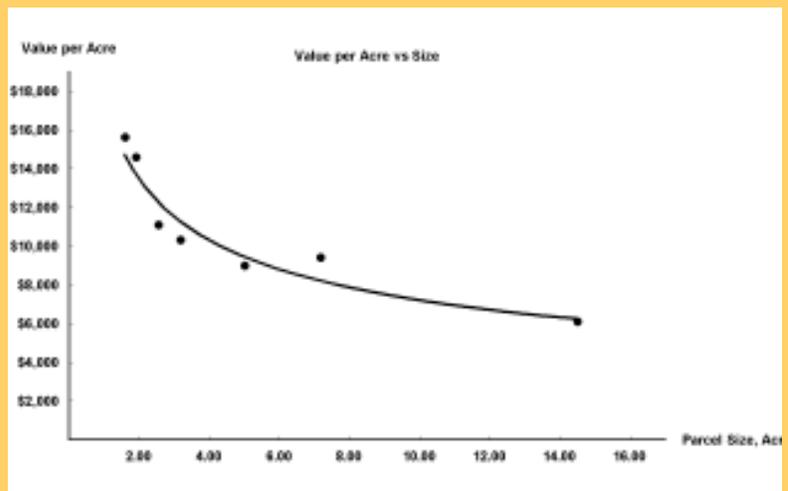
The law of decreasing returns or the principle of diminishing marginal productivity states that successive increments of one or more factors of production added to fixed amounts of other factors enhance income to a point of maximum return. In short, this principle says there is a limit that a component adds to value and as we approach that limit, the incremental contribution decreases in magnitude. As an example, a buyer may happily pay $10,000 to add one acre to his 2 acre lot (an increase of 50% in utility), but might only be willing to pay $2,000 to add that same acre to a 10 acre lot (an increase of 10% in utility).

The theory of contribution states that value of a component part of a property depends upon its contribution to the value of the whole. So, the value of any component must be determined by its value contribution to the whole and this value is limited by the value of the whole. This is why in many markets the cost for a four car finished garage

only returns about half of what it costs in a market where two car garages are the norm. This is also why the 4,500 square foot house in an area of 2,400 square foot homes also has a market value contribution of that extra area that is less than its cost.

Regression: Making Better Adjustments

If our job as an appraiser is to replicate market behavior, how can we quantify

this variable behavior in an appraisal? From the appraisals I have reviewed, too many appraisers ignore the magnitude of the independent variable (most often, size) when adjusting for the dependant quantity (size adjustment, usually a value per square foot). Those that do recognize a difference usually apply a variable adjustment based on subjective experience. The latter is certainly better than the first, but there is a better way.

The key to making better adjust-

Grid 1

	Sale 1	Sale 2	Sale 3	Sale 4	Sale 5	Sale 6	Sale 7
Price	$25,000	$28,000	$28,300	$32,900	$45,000	$67,500	$88,500
Date	Oct-99	Nov-99	Jun-99	Jun-99	Jun-99	Jun-99	Mar-99
Area/Acres	1.6	1.92	2.55	3.19	5.01	7.18	14.50
Value per acre	$15,625	$14,583	$11,098	$10,313	$8,982	$9,401	$6,103

Graph 1

ments for components that vary is to not adjust for these at all. Instead, the appraiser should design the analysis to yield an expression of the desired quantity. Then apply a regression analysis using readily available tools to extract the relationship and apply it to your subject.

Let's work through an example by extending the prior land analysis. We will extract the curved relationship for these land sales, and allow the computer to extract the formula for the curve[11], as shown in the example (Graph 2) to the right.[12]

What we've done is a regression analysis. Not only can the computer extract the formula, but we can also get the R^2 statistic – a measure of the accuracy of the equation. The formula is a curved relationship and a power function seems to fit this data the best (other functions are available). The R^2 statistic is the coefficient of determination and is an expression of the variation of the data from the regression line and falls somewhere between 0 and 1. A value near 0 indicates little correlation while a value near 1 connotes a strong relationship. For this analysis, we have an indicated R^2 of .9223 which is quite good. In real estate practice, they generally fall between 0.60 and 1.00.

Regression for Site Values

Regression is a great tool[13] for site value analysis since we know the value per acre or value per square foot is certainly not constant.

Example: We need to estimate the value of a 10 acre parcel using the results of the regression analysis above. First, manually estimate the value from the graph. Entering the graph at 10 acres along the x axis for the independent variable, read up to the curve, then over to the value per acre. This appears to be about $7,200 per acre resulting in $72,000 for the parcel.

Using the results of the prior analysis, this can be calculated more precisely. The formula for the value per acre is taken from the graph:

Graph 2
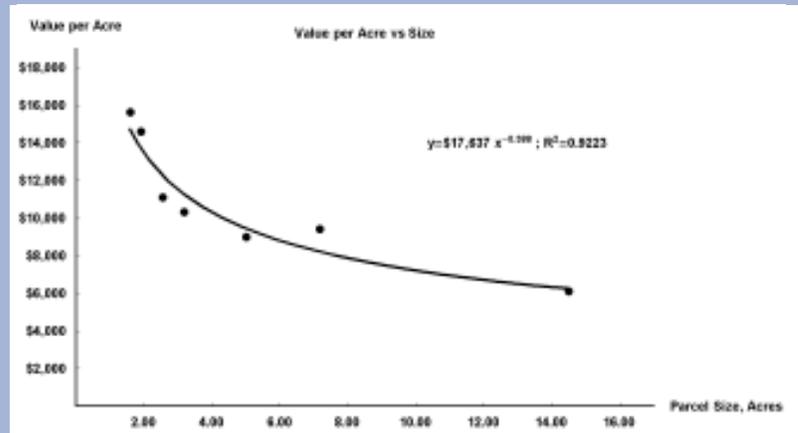
Value per Acre = $(10^{-0.388})$ x 17637
Value per Acre = $7,218

And the parcel value:

Parcel Size, Acres: 10.00
Value per Acre: $7,218
Parcel Value: $72,181

In practice, of course, we would have applied adjustments for appreciation, location, view, utilities, etc. to our sales before calculating the value per acre and processing the regression.

Although this procedure may seem backwards, it would be hard to argue that the results are not an accurate expression of the market. It certainly beats attempting to estimate size adjustments with the comps used in this analysis. Using regression analysis, we've been able to provide a bridge of accuracy between otherwise dissimilar comps.

Extending Site Value Applications

With this simple technique, you now have an easy way to estimate site values not only for your subject, but also for all the site values for each of your improved sales. You can quickly calculate actual site differences between the subject and comparables instead of applying a

constant value per acre adjustment.

The data sometimes tells interesting stories. You may find two or more trends displayed on one graph, especially for unadjusted data. Using a number of sales, I have seen these separate trends represent view parcels and non-view parcels on the same graph. Not only can you get the relationship for each segment, but you can also calculate the view premium between the two sets.

For a cluster of sales with a limited range in parcel size, your graph may appear linear for this small range. Similarly, if you display a limited range of a larger curve, this portion may also appear linear. The linear portion simply means you have an easier calculation, but be careful not to extend your results beyond your data range.

The graph of site values also becomes quite useful in finding the terminal limit of the value per acre at the upper end of parcel size. This limit is an expression of the marginal contribution of excess land – something you may need to estimate in a boundary line adjustment, land swap, or analysis of a landlocked parcel.

One of the most useful applications of a site value regression model is for subdivision appraisals. All we

need to do is to generate this one value relationship and we have essentially estimated the site values for all the lots of any size. This assumes everything else is equal – sometimes post regression adjustments for views, water frontage, access, etc. are sometimes necessary.

Market Grids: Analyze for Size - Don't Adjust

Let's construct a market approach analysis for a motel where we don't adjust for size or number of units. Instead, the value per square foot and the value per unit are the focus of the analysis and used to construct a value relationship that is then applied to the subject.

The subject we are appraising is a motel with 23 units and 12,173 square feet of finished space. Four sales were identified and used in the market analysis. The market is stable, so there is no time adjustment and there were no sales concessions for any of the sales.

Adjustments are made to each sale for location, condition, quality, owner's quarters, site value and effective age. These adjustments are added to the sales price for an adjusted value for each sale. *No adjustment is made for size or number of*

Graph 4
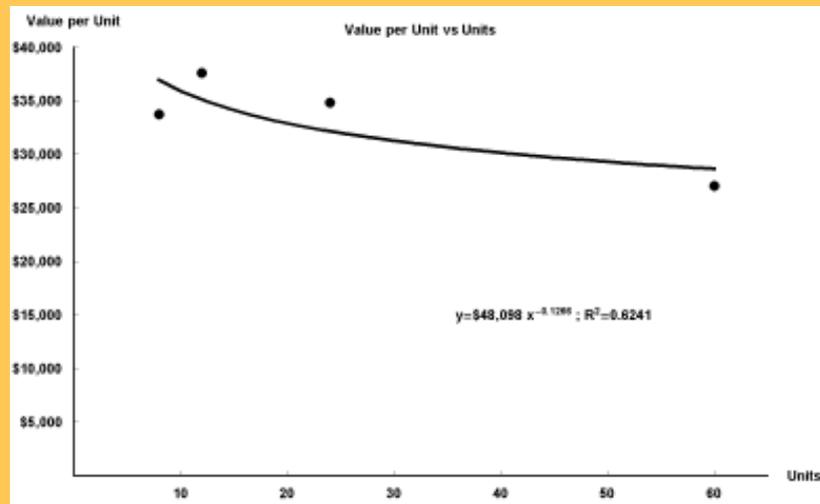
units.

The next step is to divide the adjusted value by both the area and number of units. This will yield a value per square foot for each sale as well as a value per unit. The market grid, adjustments, and resulting values are shown in Table 1 on page 19. Next, we construct a graph (Graphs 3,

Graph 3

below & 4, above) for each parameter as shown below. For each graph, we have extracted the regression formula. Note that the R^2 statistic for the value per square foot is 0.9928, indicating an excellent result with little variation. The R^2 statistic for the value per unit is only 0.62, but still a useful indicator of value (This will be noted in your reconciliation).

The final step is to apply each regression formula to the subject and reconcile the indications.

The table to the right (Table 2) shows the independent variables for the subject at 12,173 square feet and 23 units. The dependant variable is calculated from the extracted regression formulas and is shown in the next column. Multiplying yields the indicated value for each parameter. The indicated values are within a close range.

The indicators are then reconciled. More weight is usually given to the indicator which has the highest R^2 statistic, but other factors can influence the reconciliation.

Other Regression Applications

This regression procedure can be easily applied to many real estate problems that you encounter where the value per unit changes. On income properties, it is
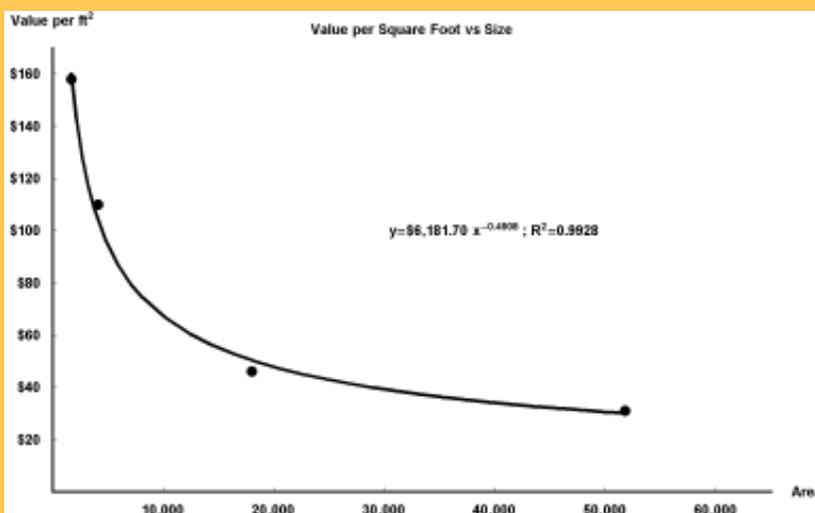
Using an analysis procedure that focuses on the variable quantity, regression analysis can be easily applied to many appraisal problems resulting in much more accurate estimates of value allowing the confident use of comparables with dissimilar sizes.

Table 1

		Sale 1	Sale 2	Sale 3	Sale 4
Sales Price		$240,000	$325,500	$1,355,533	$850,000
Date	17-Dec-99	Jun-99	May-98	Sep-99	Oct-98
Apprec.	0%	$0	$0	$0	$0
Concessions		$0	$0	$0	$0
		$240,000	$325,500	$1,355,533	$850,000
Location	Average	Average	Average	Average	Average
Cond/Qual	Good/Avg	Avg/Avg	Infr/Infr	Good/Avg	Good/Avg
Owner's Quarters	Hs/Office	Hs/Ofc/Grg	Hs/Office	Restr/Mtg Rm	Gd.House
Site:	7.5 Ac	3 Ac	4.7 Ac	22 Ac	10 Ac
Effective Age, Yrs	14	17	23	8	16
Location		$0	$0	$0	$0
Cond/Qual		$15,000	$28,784	$621,360	$0
Owner's Quarters		($15,000)	$0	$0	($45,000)
Site:		$12,000	$24,000	($150,000)	($12,000)
Eff. Age, yrs.		$18,000	$73,238	($203,330)	$42,500
Net Adjustments		$30,000	$126,022	$268,030	($14,500)
Net Adjusted Value		$270,000	$451,522	$1,623,563	$835,500
Area:	12,173	1,712	4,112	51,780	18,035
Value/SF		$158	$110	$31	$46
Units:	23	8	12	60	24
Value/Unit		$33,750	$37,627	$27,059	$34,813

insightful to analyze three ratios with a regression analysis: value per unit, value per room, and value per gross income. A graph of each is prepared and results are calculated and reconciled.

Regression analysis can be quite helpful in estimating market rents[14], especially with a wide range of sizes or limited data. An analysis of rents using this procedure with a graph of the rent per square foot versus the size of the space will usually yield a useful graph of the relationship that can be used to estimate the subject rent.

In situations where indicators are unknown or questionable, you can allow the data to demonstrate its own relationships. The mathematically adventurous can can apply a multiple regression analysis for several factors and rather quickly determine what indicators are significant

Table 2

	Subject	Regression Indicator	Indicated Value
Area:	12,173	$61.45	$747,979
Units:	23	$32,850	$755,546

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[7] Robert J. West, MAI, "Subdivision Analysis: Retail Values", The Appraisal Journal (1989), v.LVII/2.

[8] To prepare this graph in Microsoft Excel (Office 97 or 2000), highlight the data for acres and value per acre on the spreadsheet, click on the graph icon on the tool bar, click on Scatter, click next, click Finish. (These instructions give you the basics - enhancements are possible)

[9] Appraisal Institute, The Appraisal of Real Estate, 11th ed. (Chicago: Appraisal Institute, 1996), pp.44-45.

[10] Appraisal Institute, The Appraisal of Real Estate, 11th ed. (Chicago: Appraisal Institute, 1996), p.45.

[11] A power relationship seems to give the best results in most cases and yields results that are easiest to work with as opposed to exponential or logarithmic relationships.

[12] Using Microsoft Excel (Office 97 or 2000), right click on any data point on the graph, click on Add Trend line, click on Power, click on Options, click the formula and R^2 boxes, click OK.

[13] David J. January, SRA, MAI, "Forecasting Lot Values Using Regression Analysis", The Real Estate Appraiser & Analyst (1979-1990); v55/3.

[14] Stephen C. Kincheloe, MAI, "Linear Regression Analysis of Economic Variables in the Sale Comparison and Income Approaches", The Appraisal Journal (1993), v61/4.

(R^2 statistic) and the relationship for each of those indicators. For instance, I have found that convenience stores have a well defined value relationship that can be expressed as a function of their site size, building size, petroleum sales and merchandise sales.

Conclusion

Many factors which naturally occur in real estate invite more thorough analysis because the values change depending on their quantity. Using an analysis procedure that focuses on the variable quantity, regression analysis can be easily applied to many appraisal problems resulting in much more accurate estimates of value allowing the confident use of comparables with dissimilar sizes. Most appraisers already have the necessary computers and software and are only a few clicks away from greatly improving the quality of their appraisals.

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